



Topic: Annuities and Perpetuities

Time: 45 mins

Marks: /45 marks

Calculator Assumed

Question One: [3, 2, 2: 7 marks]

Once reaching retirement age Betty draws on her superannuation and invests it into an account paying interest 7.3% p.a. compounding annually. She will withdraw \$47 000 after one year and after each year thereafter. Betty has \$520 000 in her superannuation upon retirement.

- a) Describe the recursion rule for the amount left in the investment at the start of each year.

- b) How much is left in the investment account at the start of the 14th year?

Betty retires at age 65 and she wants this investment to continue paying a regular, constant withdrawal each year until she is at least 90 years old.

- c) If the investment is to last until she is at least 90 years old, what is the most she can withdraw each year?

Question Two: [3, 2: 5 marks]

Granny Goldie always gives each of her four children \$50 their respective birthdays.

- a) How much does she need to invest into an account which pays 4.5% interest p.a. compounding quarterly in order for her to withdraw \$50 each quarter perpetually.

- b) If the interest rate is still compounding quarterly but she wants to take out all the money for the year in one transaction, does this change how much she needs to invest?

Question Three: [2, 1, 4, 3: 10 marks]

Keith's parents invested \$500 into an investment account for him when he was born. The account was paying interest at 6.5% p.a for a fixed term of 18 years. The account remained untouched until his 18th birthday, when Keith withdraws all the money.

- a) How much money does Keith get on this 18th birthday?

Keith adds \$2500 of his own savings to his 18th birthday money, rounds it to the nearest \$10 and invests it in an account paying 8.3% interest p.a compounding monthly. Keith wants to withdraw \$100 from this investment after the first month and he to continue withdrawing but to increase his withdraws by 20% each month.

- b) How much is Keith's initial investment?
- c) Show the recursive rule for the amount of money left in the account at the end of each month.
- d) How long can Keith continue to make withdrawals and what will be the amount of his final withdraw?

Question Four: [2, 3, 3, 3: 11 marks]

The Shire of Perth wants to create a perpetual award for an outstanding young person who gives of themselves to the community. They wish for the award to include an education grant of \$900. They have four investment account options, how much would they need to invest in each account in order to create this annual perpetual award.

a) 7.0% p.a compounding annually.

b) 6.2% p.a compounding monthly.

c) 3.2% p.a compounding daily.

d) 5% p.a compounding bi-annually.

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Question Five: [2, 2, 2: 6 marks]

\$500 is invested into an account paying 5% interest p.a. Describe the shape of the graph depicting the balance in the account at the end of each year if:

- a) \$40 is withdrawn from the account at the end of each year.

- b) \$25 is withdrawn from the account at the end of each year.

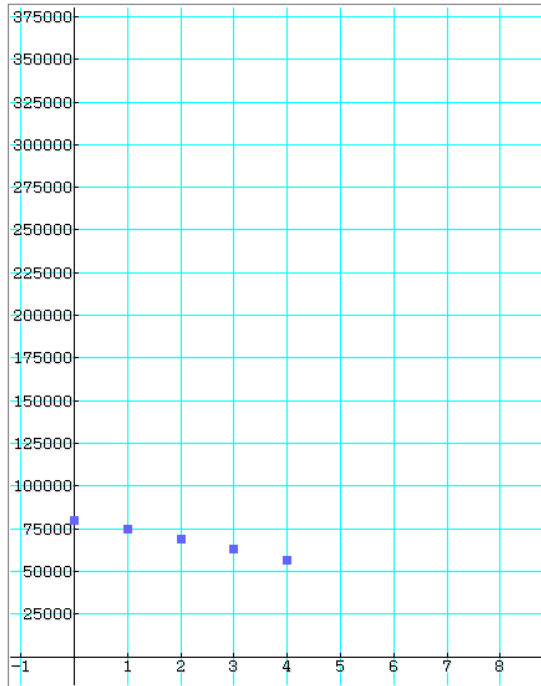
- c) \$10 is withdrawn from the account at the end of each year.

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Question Six: [2, 2, 2: 6 marks]

Carol goes into semi-retirement 5 years before she can access her superannuation. She has \$80 000 in an investment account which is earning 5.83% interest p.a compounding annually and she draw \$10 000 from this account at the end of each year.

The following graph shows the balance in the account at the end of each year for the first four years.



By the end of the 5th year Carol is given access to her superannuation and \$200 000 is added to this account.

- Show the balance at the end of the 5th year on the graph above.
- If Carol now begins to withdraw \$60 000 at the end of each year, show the recursive formula to describe the balance in the account at the end of each year.
- Is the situation described in part b) a satisfactory plan for Carol's retirement? Explain your answer with reasoning.



Topic: Annuities and Perpetuities
SOLUTIONS

Time: 45 mins

Marks: /45 marks

Calculator Assumed

Question One: [3, 2, 2: 7 marks]

Once reaching retirement age Betty draws on her superannuation and invests it into an account paying interest 7.3% p.a. compounding annually. She will withdraw \$47 000 after one year and after each year thereafter. Betty has \$520 000 in her superannuation upon retirement.

- a) Describe the recursion rule for the amount left in the investment at the start of each year.

$$T_{n+1} = T_n \times 1.073 - 47\,000 \quad T_0 = 520\,000$$

- b) How much is left in the investment account at the start of the 14th year?

$$\$334\,348.94$$

Betty retires at age 65 and she wants this investment to continue paying a regular, constant withdrawal each year until she is at least 90 years old.

- c) If the investment is to last until she is at least 90 years old, what is the most she can withdraw each year?

$$\$45\,833.98$$

Question Two: [3, 2: 5 marks]

Granny Goldie always gives each of her four children \$50 their respective birthdays.

- a) How much does she need to invest into an account which pays 4.5% interest p.a. compounding quarterly in order for her to withdrawal \$50 each quarter perpetually.

$$\frac{4.5}{4} = 1.125\% \text{ per quarter}$$

$$A \times 0.01125 = 50$$

$$A = \$4\,444.45$$

- b) If the interest rate is still compounding quarterly but she wants to take out all the money for the year in one transaction, does this change how much she needs to invest? **Yes she could invest slightly less as she will see a \$3.40 growth per year on her current investment.**

Question Three: [2, 1, 4, 3: 10 marks]

Keith's parents invested \$500 into an investment account for him when he was born. The account was paying interest at 6.5% p.a for a fixed term of 18 years. The account remained untouched until his 18th birthday, when Keith withdraws all the money.

- a) How much money does Keith get on this 18th birthday?

$$500 \times 1.065^{18} = \$1\,553.33 \checkmark \checkmark$$

Keith adds \$2500 of his own savings to his 18th birthday money, rounds it to the nearest \$10 and invests it in an account paying 8.3% interest p.a compounding monthly. Keith wants to withdraw \$100 from this investment after the first month and he to continue withdrawing but to increase his withdraws by 20% each month.

- b) How much is Keith's initial investment?

$$\$4\,050 \checkmark$$

- c) Show the recursive rule for the amount of money left in the account at the end of each month.

$$T_{n+1} = T_n \times 1.00691\dot{6} - 100 \times 1.2^n \quad T_0 = 4\,050 \quad \text{or} \quad T_{n+1} = \left(1 + \frac{8.3}{1200}\right) T_n - (1.2)^n \times 100$$

$\checkmark \quad \checkmark \quad \checkmark \quad \checkmark$

- d) How long can Keith continue to make withdrawals and what will be the amount of his final withdraw?

\checkmark

He can continue to withdraw for 13 months.

His final withdrawal is $344.06 \times 1.00691\dot{6} = \346.44

$\checkmark \quad \checkmark$

Question Four: [2, 3, 3, 3: 11 marks]

The Shire of Perth wants to create a perpetual award for an outstanding young person who gives of themselves to the community. They wish for the award to include an education grant of \$900. They have four investment account options, how much would they need to invest in each account in order to create this annual perpetual award.

- a) 7.0% p.a compounding annually.

$$A \times 0.07 = 900 \quad \checkmark$$

$$A = \$12\,857.14 \quad \checkmark$$

- b) 6.2% p.a compounding monthly.

$$A \times \left(1 + \frac{0.062}{12}\right)^{12} = A + 900 \quad \checkmark \quad \checkmark$$

$$A = \$14\,108.23 \quad \checkmark$$

- c) 3.2% p.a compounding daily.

$$A \times \left(1 + \frac{0.032}{365}\right)^{365} = A + 900 \quad \checkmark \quad \checkmark$$

$$A = \$27\,678.63 \quad \checkmark$$

- d) 5% p.a compounding bi-annually.

$$A \times \left(1 + \frac{0.05}{2}\right)^2 = A + 900 \quad \checkmark \quad \checkmark$$

$$A = \$17\,777.78 \quad \checkmark$$

Question Five: [2, 2, 2: 6 marks]

\$500 is invested into an account paying 5% interest p.a. Describe the shape of the graph depicting the balance in the account at the end of each year if:

a) \$40 is withdrawn from the account at the end of each year.

Graph is declining since the balance is going down. ✓ ✓

b) \$25 is withdrawn from the account at the end of each year.

Graph is a horizontal line since \$25 is exactly 5% of \$500. ✓ ✓

c) \$10 is withdrawn from the account at the end of each year.

Graph is increasing because the amount being withdrawn is less than the interest being added.

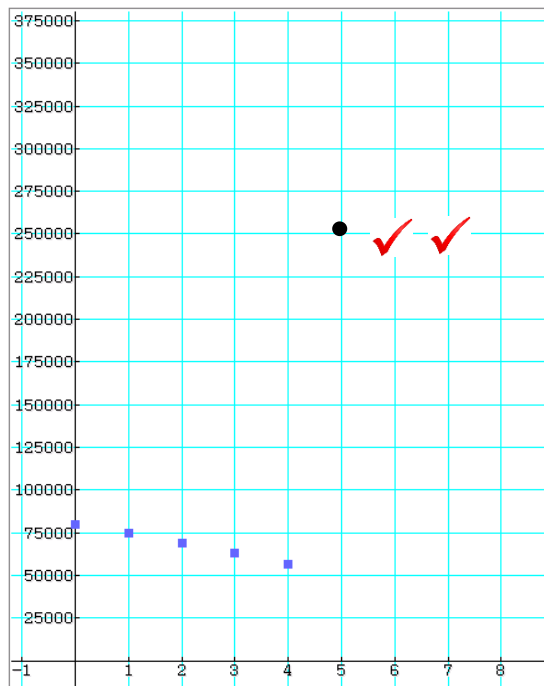
✓ ✓

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Question Six: [2, 2, 2: 6 marks]

Carol goes into semi-retirement 5 years before she can access her superannuation. She has \$80 000 in an investment account which is earning 5.83% interest p.a compounding annually and she draw \$10 000 from this account at the end of each year.

The following graph shows the balance in the account at the end of each year for the first four years.



By the end of the 5th year Carol is given access to her superannuation and \$200 000 is added to this account.

- Show the balance at the end of the 5th year on the graph above. **\$250 022**
- If Carol now begins to withdraw \$60 000 at the end of each year, show the recursive formula to describe the balance in the account at the end of each year.

$$T_{n+1} = T_n \times 1.0583 - 60\,000 \quad T_0 = 250\,022$$



- Is the situation described in part b) a satisfactory plan for Carol's retirement? Explain your answer with reasoning.

No it isn't because she will run out of money by the 5th year after accessing her super funds. 5 years isn't a long enough retirement plan.

